Worksheet: Oceans on the rise

Evidence for rising sea levels

The potentially dangerous side effects of global warming like rising sea levels are not something that will happen sometime in the future. It is already happening. Sea levels are constantly monitored via oceanic probes and remote sensing from earth observing satellites.

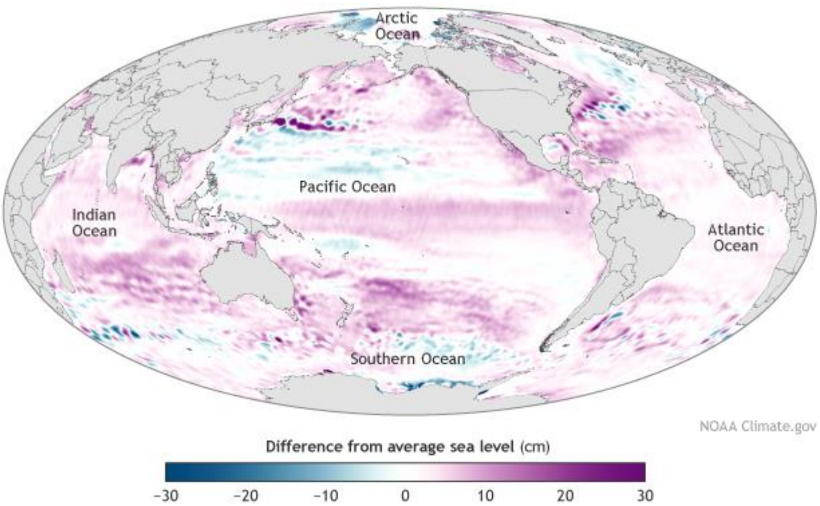

Figure 1: Sea level in 2014 compared to the global average at the mid-point of the 1993-2013 time series (NOAA Climate.gov map, adapted from Figure 3.25a in State of the Climate in 2014 report, https://www.climate.gov/news-features/understanding-climate/2014-state-climate-sea-level).

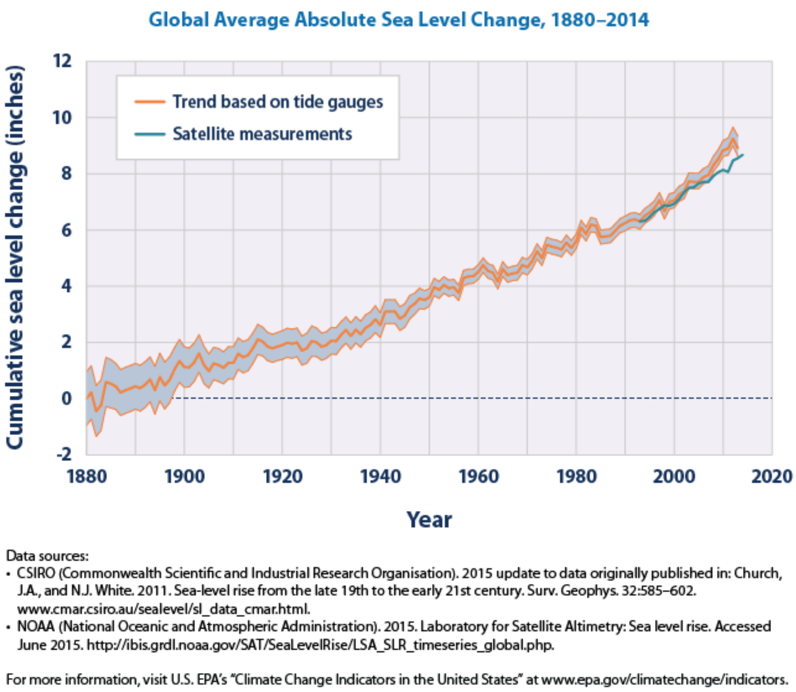

Figure 2: This graph shows cumulative changes in sea level for the world's oceans since 1880, based on a combination of long-term tide gauge measurements and recent satellite measurements. This figure shows average absolute sea level change in inches (1 inch = 25.4 mm), which refers to the height of the ocean surface, regardless of whether nearby land is rising or falling. Satellite data are based solely on measured sea level, while the long-term tide gauge data include a small correction factor because the size and shape of the oceans are changing slowly over time. (On average, the ocean floor has been gradually sinking since the last Ice Age peak, 20,000 years ago.) The shaded band shows the likely range of values, based on the number of measurements collected and the precision of the methods used (Credit: United States Environmental Protection Agency, https://www3.epa.gov/climatechange/science/indicators/oceans/sea-level.html).

Several processes contribute to this phenomenon. One of them is the thermal expansion of water. This is called the thermosteric component of the sea level change.

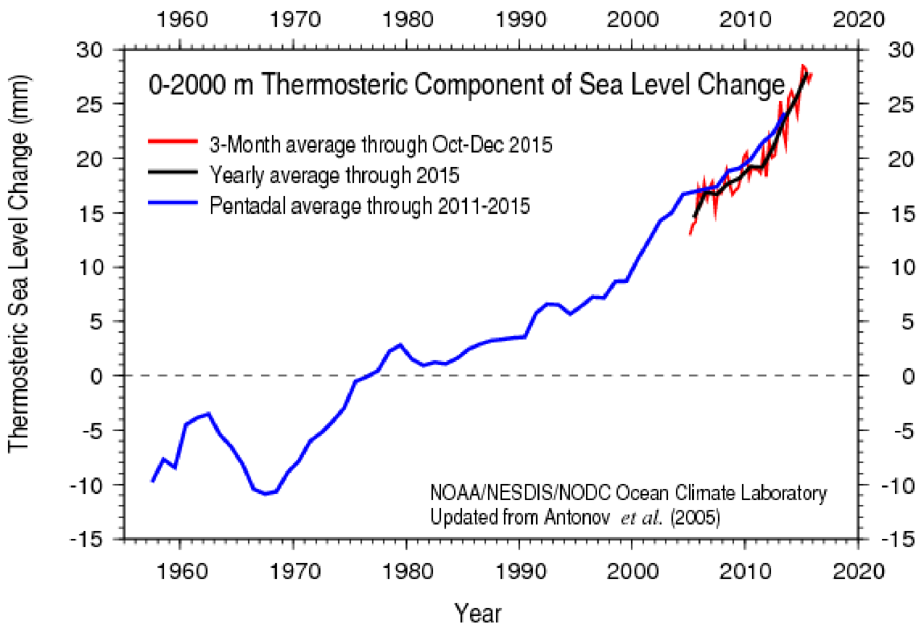

Figure 3: Cumulative temporal evolution of the sea level change caused by heat expansion (thermosteric sea level change) between 1955 and 2015 (http://www.nodc.noaa.gov/OC5/3M_HEAT_CONTENT/).

The physics of thermal expansion

When heated, liquids expand isotropically. This is the principle of the common thermometer. With a constant pressure and a constant amount of substance, the relative volume increase $\frac{\Delta V}{V}$ is sufficiently described by the following linear differential equation.

$$\gamma = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p,N}$$

The quantity γ is the so called cubic expansion coefficient with the unit K^{-1}. If γ does not change with temperature, we can write $\gamma(T) = \gamma(T_0)$, and the linear differential equation can easily be solved. This is actually incorrect, but a good approximation for small temperature changes.

$$V(T) = V_0 \cdot \exp(\gamma \cdot \Delta T)$$

For small temperature variations, a Taylor expansion results in:

$$V \approx V_0 + V_0 \cdot \gamma \cdot \Delta T = V_0 \cdot (1 + \gamma \cdot \Delta T)$$

Or in other terms:

$$\Delta V = V_0 \cdot \gamma \cdot \Delta T$$

With these properties, the temperature dependence of density can be explained. As the mass does not depend on the temperature, the following equation is valid:

$$m = \varrho \cdot V = \text{const.}$$

With m being the constant total mass. Since the volume V changes with temperature, the density ϱ does as well. In reality, the cubic expansion coefficient is a function of temperature. Therefore, the actual value must be selected, even if the variations around the given temperature are small.

Values of the temperature dependent density of water are summarised in Table 1.

Table 1: The density of water for various temperatures.

Temperature ϑ in °C	Density ϱ in $\frac{\mathrm{g}}{\mathrm{dm}^3}$	Temperature ϑ in °C	Density ϱ in $\frac{\mathrm{g}}{\mathrm{dm}^3}$
0	999.84	16	998.94
1	999.90	17	998.77
2	999.94	18	998.59
3	999.96	19	998.40
4	999.97	20	998.20
5	999.96	21	997.99
6	999.94	22	997.77
7	999.90	23	997.54
8	999.85	24	997.29
9	999.78	25	997.04
10	999.70	26	996.78
11	999.60	27	996.51
12	999.50	28	996.23
13	999.38	29	995.94
14	999.24	30	995.64
15	999.10	31	995.34

Activity 1: Experiment

We will investigate the effect of thermal expansion of water.

List of material

- Flask
- Rubber stopper with a hole
- Glass tube with small diameter (or capillary tube)
- Strong lamp
- Thermometer
- Overhead marker (water based)

Experimental set-up

1. The flask is completely filled with water.
2. Measure the temperature of the water with a thermometer.
3. Close the tube including the glass tube or capillary tube with the stopper avoiding air bubbles. The water level in the glass tube should be visible.
4. Mark the water level on the glass tube with the overhead marker.
5. Position the lamp at a distance of approx. 5 cm in front of the flask.

Figure 4: Experimental set-up.

Procedure

Switch on the lamp and observe the liquid in the glass tube for several minutes.

Tasks and Questions

- Write down your observations.
- Measure the temperature at the end of the experiment.
- By how much did the water rise in the tube?
- By how much did the temperature change?
- What conclusions can be drawn for the Earth especially for the oceans?

Activity 2: Calculations

Exercise 1: Expansion of water

1 litre (=1 dm^3) of water is heated from 10°C to 20°C.

- What is the volume of the water after warming up? Note that the total mass of the water does not change.
- What is the increase of the volume in percent?
- Use the proportionality between the density of water and its volume for different temperatures. The volume has to increase at the same rate as the density decreases.

Exercise 2: Sea level rise of an ideal ocean

We assume that the average depth of the oceans is 3,800 m and its mean temperature is 5°C.

- What is the volume of a water column in dm^3 of a cuboid with a base area of 1m^2 and a height of 3,800 m?
- Provided the temperature of the ocean would have an average of 5°C. What would be the volume of the water after a temperature increase of 1°C?
- By how much does the sea level rise?

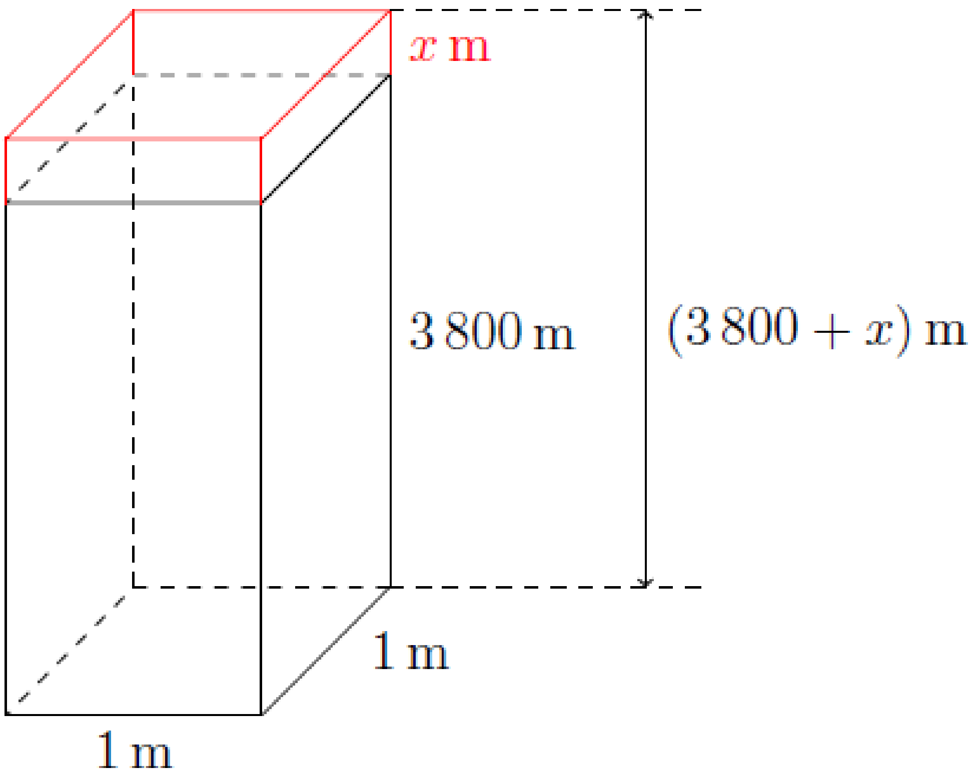

Figure 5: Sketch of a water column with a square base area of 1 m x 1 m and an altitude of 3800 m.

What does that mean for the rise of the sea levels?

Solutions

Activity 1: Experiment

Due to the illumination with the lamp the temperature of the water increases and the water level in the glass tube rises, i.e. it expands. Liquids generally expand with increasing temperature.

Activity 2: Calculations

Exercise 1: Expansion of water

$$m = \varrho \cdot V = \varrho_{10} V_{10} = \varrho_{20} V_{20}$$
$$\Leftrightarrow V_{20} = \frac{\varrho_{10}}{\varrho_{20}} V_{10}$$

The values for the densities are taken from the table.

$$V_{20} = \frac{999.70 \frac{\mathrm{g}}{\mathrm{dm}^3}}{998.20 \frac{\mathrm{g}}{\mathrm{dm}^3}} \cdot 1\ell \approx 1.0015\ \ell$$

What is the increase of the volume in percent?

$$\frac{V_{20} - V_{10}}{V_{10}} = \frac{1\ell - 1.0015\ \ell}{1\ \ell} = 0.0015 = 0.15\%$$

The volume has increased by 0.15%.

Exercise 2: Sea level rise of an ideal ocean

$$V = 1\ \mathrm{m} \cdot 1\ \mathrm{m} \cdot 3800\ \mathrm{m} = 3800\ \mathrm{m}^3$$

$$V_6 = \frac{\varrho_5}{\varrho_6} V_5 = \frac{999.96 \frac{\mathrm{g}}{\mathrm{dm}^3}}{999.94 \frac{\mathrm{g}}{\mathrm{dm}^3}} 3800\ \mathrm{m}^3 = 3800.076\ \mathrm{m}^3$$

The change in volume is:

$$\Delta V = V_6 - V_5 = 0.076\ \mathrm{m}^3$$

If we consider the area of the base of the water column $A = 1\ \mathrm{m} \cdot 1\ \mathrm{m} = 1\ \mathrm{m}^2$, the change of height Δh amounts to:

$$\Delta h = \frac{\Delta V}{A} = \frac{0.076\ \mathrm{m}^3}{1\ \mathrm{m}^2} = 0.076\ \mathrm{m} = 7.6\ \mathrm{cm}$$

The rise of the sea level due to thermal expansion is a rather small yet measurable contribution to the overall phenomenon.