“How to estimate the Magnitude of an earthquake”

This lesson plan was developed according to the Italian national curriculum

Introduction:

On July 2016, a seismic network was installed in the headquarters of the partners’ organizations of the Erasmus + Schools Study Earthquakes (SSE) project. The network is composed by five TC1 type electro-magnetic seismometers provided by the National Observatory of Athens - the leading organization of the project - placed respectively in Athens, Sofia, Nicosia, Izmir and Naples. During the first half of 2017, more seismometers will be installed in some high schools in these countries.

On August 24th, a seismic crisis in central Italy caused victims and damages. The main shocks occurred from August to October were well recorded by all the seismometers of the SSE’s network. Moreover, the seismometer installed in Città della Scienza (the headquarter of Fondazione Idis), being the closest to the epicentres, recorded also the lighter aftershocks occurred after the main ones. All these shocks occurred 200 -250 km away from this seismic station, hence their recordings could be the basic tool to develop this lesson plan for the 12th and 13th school grades, that focuses on the concept of Magnitude and its physical mean.

Introduzione:

Schools Study Earthquakes is an European project (Erasmus+ programme, measure KA2 - Cooperation for Innovation and the Exchange of Good Practices) aimed to develop and implement good practices in the field of scientific education in schools with particular reference to Earth’s sciences and seismology. SSE began on September 2015 and finished on August 2017.

On July 2016, in the framework of the the first core of a seismic network has been realized by installing educational seismometers in the headquarters of the partner organizations of the project, respectively in Athens, Izmir, Nicosia, Sofia and Naples. During the first half of 2017 further seismometers will be temporarily installed in the headquarters of several schools involved in the activities of the project, in the same countries of the partner organizations. The pupils of the involved schools will be trained in using these seismometers, and in extracting, elaborating and analyzing of gathered data.

To realize the seismic network they TC1 (one vertical component seismometers) provided by the National Observatory of the Athens, the organization leading the project, have been utilized. The TC1 seismometers are conceived on purpose of educational tasks and med with simple materials, however they are able to reveal the ground vibrations due to the passage of seismic waves generated by both low local earthquakes or strong earthquakes generated very far away.

From August 24th 2016 a dramatic seismic crisis started striking central Italy and causing victims and damages. The crisis continued until the next October reaching its climax with with a Magnitude 6.5 shock occurred on 30th.

The strongest shocks occurred during the crisis were been clearly recorded by all the seismometers of the SSE network, while the one installed in Città della Scienza, the headquarter of Fondazione Idis, managed to record clearly also several lower Magnitude aftershocks, being the closest to their epicentres. All the shocks of the seismic crisis occurred at an epicentral distance ranging from 200 to 250 km from Città della Scienza.

The gathered data could represent the basis to develop a lesson plan upon the idea of Magnitude, its physical mean and its estimation. The lesson plan could be aimed to the students of 11^{th} – 13^{th} degree.

The concept of Magnitude was introduced on 1935 by Charles Richter in collaboration with Beno Gutemberg on order to define unambiguously the size of an earthquake referring to the amount of energy released by the earthquake itself, this in analogy to the concept of Magnitude used by the astronomers to classify the stars based on their brightness. The Magnitude of an earthquake is obtained by the maximum amplitude of the oscillations of the ground detected by a standard device, and by the distance between the measuring place and the epicentre. Magnitude is expressed as a decimal logarithm in order to report in a rather narrow numerical range both barely perceptible and "giant" earthquakes: in practice, each increase of one Magnitude unit corresponds to a 10-fold increase in the maximum amplitude shown on the seismogram, and a 30-fold greater release of energy. On Richter scale earthquakes having a Magnitude lower than 2,0 are defined as "instrumental events", this means in general they aren't felt by people but they are revealed just by the seismometers closest to their epicentre. Earthquakes having Magnitude over 4,5 are strong enough to be revealed very far away from their epicentres, at least by the most sensitive devices. Finally, earthquakes having magnitude over 8.0 are defined as "giant earthquakes".

The Magnitude as defined by Richter is indicated as M_L (local Magnitude) and it is expressed as the decimal logarithm of the maximum amplitude of the waveform of an earthquake recorded by a standard Wood-Anderson[1] type seismometer installed 100 km far from the epicentre. Then, the formula of local Magnitude is:

$M_L = log_{10} A$

Where is the maximum peak in amplitude of the waveform, in micrometres.

Considering it is highly unlikely to find a seismometer installed exactly 100 km far from the epicentre, M_L could be estimated by correcting the formula above if we know the attenuation of the amplitude of seismic waves depending on epicentral distance.

Richter obtained the attenuation law for the geological formations of southern California referring to numerous recordings with epicentral distances ranging from 20 to 500 km. the data he collected could be resumed into two equations:

[1] The Wood-Anderson is a 3 components torsion seismometer in use until the 60's of XX Century. The standard calibration entails 2800 amplification factor, 0,8 *s* resonance period and 0,8 damping constant.

$M_L = log_{10} A + 1,6\ log_{10} D – 0,15$, *for epicentral distances shorter than 200 km*

$M_L = log_{10} A + 3,0\ log_{10} D – 3,38$ *for epicentral distances ranging from 200 to 600 km.*

The numerical constants of the two formulas are valid for that region of U.S.A.; A is the maximum amplitude of the waveform expressed in micrometres while D is the epicentral distance expressed in kilometres.

In any case the measure of the Magnitude is given by a generic formula like the one below:

$M = log_{10} (A/T) + f(D, h) + C_S + C_r$

Where A is the maximum displacement of the ground due to the type of waves on which the considered Magnitude is based (e.g.: fig. 1), T is the period of the considered phase (in practice, the distance in time between two consecutive peaks of the chosen type of waves), f is a correction factor depending on epicentral distance (D) and on the depth of the hypocentre (h), C_S is a factor depending on the geological features of the place where the seismic station is installed, while C_r is an analogous correction factor for the seismic source.

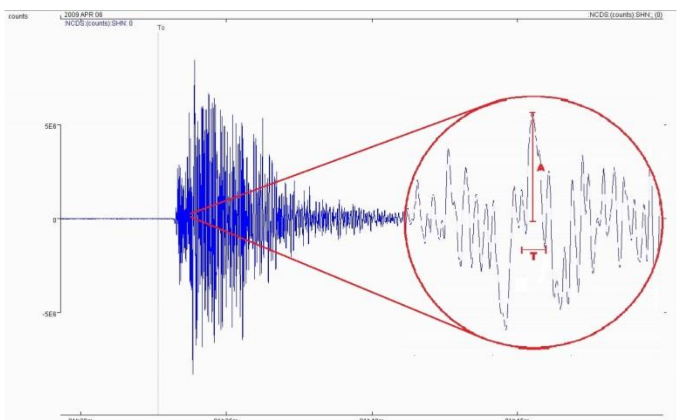

Fig. 1

In order to extend the idea of Richter to measure earthquakes coming from middle and long distances and to waveforms gathered by other kinds of seismometer characterized by other own resonance frequencies, new Magnitude scales were introduced in a way to be anyway commensurable to Richter Magnitude: body waves (mb) and surface waves Magnitudes (M_S)

Moreover, two further Magnitude scales are in use. The first is the so called momentum Magnitude or M_W, used to measure the strongest earthquakes because the maximum amplitudes of waveforms, once the M_L is over 6,5, tends to a limit value. Moreover M_W considers in addition to the movement of the ground also the energy released by the event. The second scale – Md - measures the duration of an earthquakes, instead of the maximum amplitude, and it is effective just for local earthquakes.

The calculation of local Magnitude in practice

Estimate the Magnitude of an earthquake is a particularly complicated, especially when trying it using just data gathered by an own seismometer without professional devices and competencies. In particular there are three main problems affecting this procedure:

- Systematic or random error – every seismic station measures the Magnitude affected by a systematic error which value of which can reach up to 0,3 degree more or less the true one. The extent of this error can be evaluated by comparing the Magnitudes estimated starting from waveforms gathered by this instrument with those available in the web reported in the databases of research organizations. The average of the deviations for any one of these comparisons allows us to estimate the extent of the systematic error but of course it couldn't represent a factor to correct the Magnitude. Moreover, using a 1 horizontal component sensor, the amplitude of the waveform depends on its orientation with respect to the direction of origin of the seismic waves.
- Local geological features - the factors of correction C_S and C_r appearing in the generic formula (see above) depend on the geological features of both the seismic source and the place where the seismometer is installed. For example, the correction factor once used by Richter are right for southern California but they don't work for other places in Earth' surface.
- Dependence on the sensor – the equation to calculate the Magnitude as formulated by Richter is based in results gathered by horizontal seismometers Wood-Anderson type calibrated according with established standards. Using a device having different amplification and own resonance period, the gathered Magnitudes would represent values of a scale apart, not commensurable with the one universally accepted.

The three problems above could be bypassed using a certain number of waveforms gathered with your own device[2], thus empirically determining an equation valid for your it and for the place where it is installed.

Moreover it is needed to "calibrate" your recording by reference to the Magnitudes of the same events provided by the databases of organizations operating in geophysics

As already said, the generic formula that expresses the Magnitude of a local earthquake is:

$M_L = log_{10} A + f(D) + constant$

where A is the maximum amplitude of the waveform and f(D) is a function depending on the epicentral distance. Hence, if we consider also the effects due to the features of both your own seismometer and the geology of the place where it is installed, a general complete formula is:

$M_L = log_{10} A + a \cdot log_{10} D - b$

Therefore it is necessary to determinate a and b in such generic formula. Then if we have the recording of two different earthquakes and we know their Magnitudes and their epicentral coordinates, calling

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[2] *To this task it could be used one vertical sensor or at least two horizontal sensor perpendicularly oriented to each other.*

them event 1 and event 2, we can determinate a and b by solving system of linear equations with two unknowns:

$M_{L1} = log_{10} A_1 + a \cdot log_{10} D_1 - b$

$M_{L2} = log_{10} A_2 + a \cdot log_{10} D_2 - b$

For instance, if we have the waveforms of two earthquakes with different Magnitudes and happened at different epicentral distance from the station, like in the table below:

Event	Local Magnitude (M_L)	Epicentral distance (D)	Maximum amplitude (A)
1	6,5	270 km	10 μm
2	3,6	60 km	0,25 μm

solving the system[3] we will have:

$a = 1,98$

$b = -0,69$

then

$M_L = log_{10} A + 1,98 \cdot log_{10} D + 0,69$

Therefore, in case an earthquake occurring 120 km far away from the station will provoke a maximum displacement 2,4 μm in the waveform, using this values in the referring equation we could easily calculate its local Magnitude

$M_L = log_{10} 2,4 + 1,98 \cdot log_{10} 120 + 0,69 = 5,18$

Estimate the duration Magnitde

The procedure showed above is quite simple but unreliable because even few differences of A and D could cause significant variations of the calculated Magnitude.

[3] *To solve the system, the following expressions will be adopted*
$b = log_{10}A1 + a \cdot log_{10}D_2 - M_{L1}$
then $a = (M_{L2} - log_{10}A_2 + log_{A1} - M_{L1})/(log_{10}D_2 - log_{10}D_1)$

Having only one seismic station it is possible estimate more accurately the Magnitude of an earthquake based on the duration of the ground motion recorded. Indeed it is well-known that at the same epicentral distance, the more is the magnitude the more is the duration of the recording.

To this task we can use a special function of the software Winquake. To show this procedure, we can start directly with a practical example. The picture below shows the recording gathered by the seismic station of Città della Scienza of the main shock of the earthquake of Amatrice occurred in central Italy on August 24th 2016.

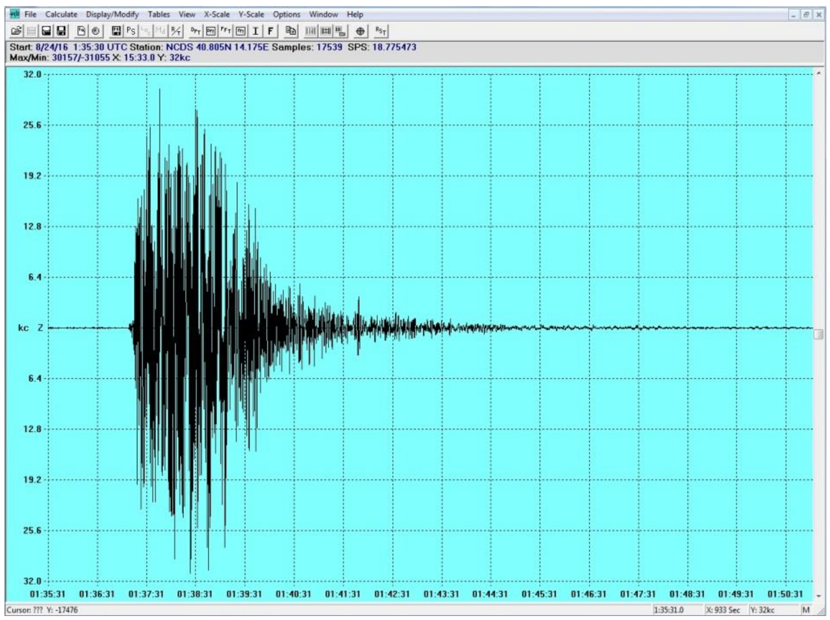

Fig. 2

As already said, at the same epicentral distance, the more is the magnitude the more is the duration of the recording.

Therefore to estimate the Magnitude of this earthquake we need first to determinate the epicentral distance. Hence we should dilate the waveform by the X-scale function in order to better visualize the first arrivals of P and S waves, and pick them using the P S function. The epicentral distance will appear at the top of the screen – as showed in the picture – that in this case corresponds to about 225 km.

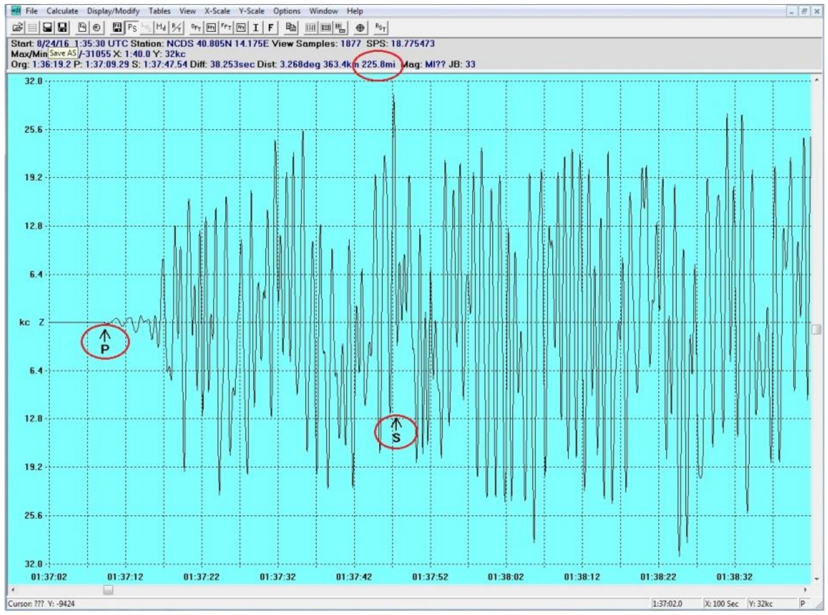

Fig. 3

At this point we can compress again the waveform to view it as a whole always using X-scale. Then we can activate the function M_d and the D mark point will appear allowing us to pick the end of the waveform. The D marc point should be placed where the amplitude of the tail of the waveform will be comparable to the amplitude of the noise. At the top of the screen will appear also M_d that in this case will correspond to 6,0 as estimated also by the researchers of the Italian INGV (see picture).