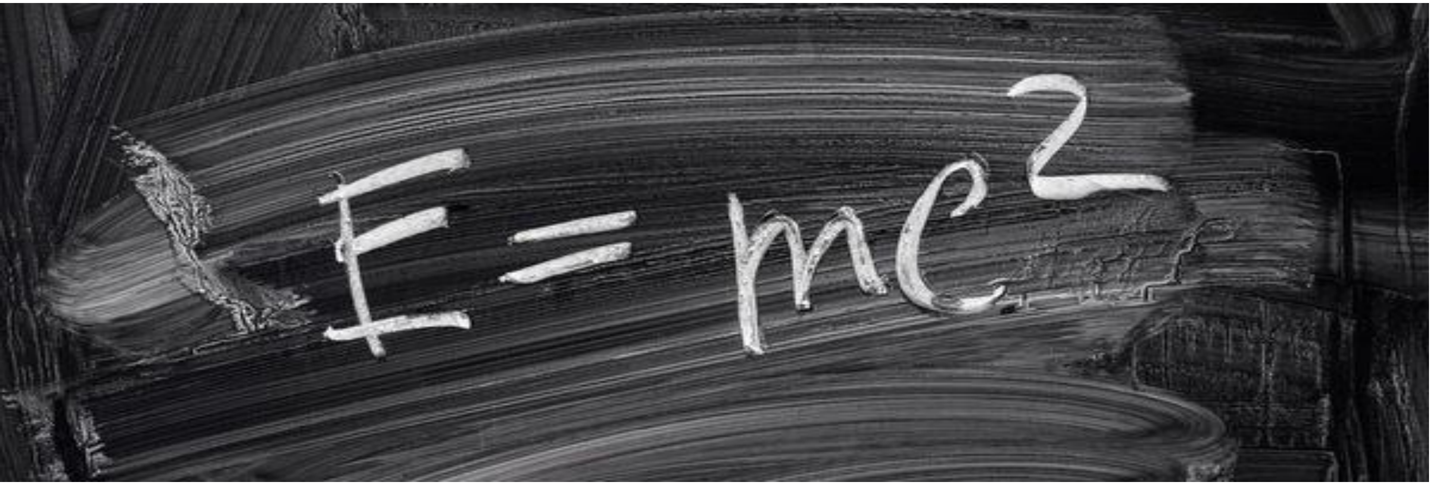

Does a body's Inertia depend upon its Energy Content?

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In the following, we shall describe a simplified version of Einstein's first proof on E=mc2
as described in his seminal paper in 1905
https://www.fourmilab.ch/etexts/einstein/E_mc2/e_mc2.pdf .

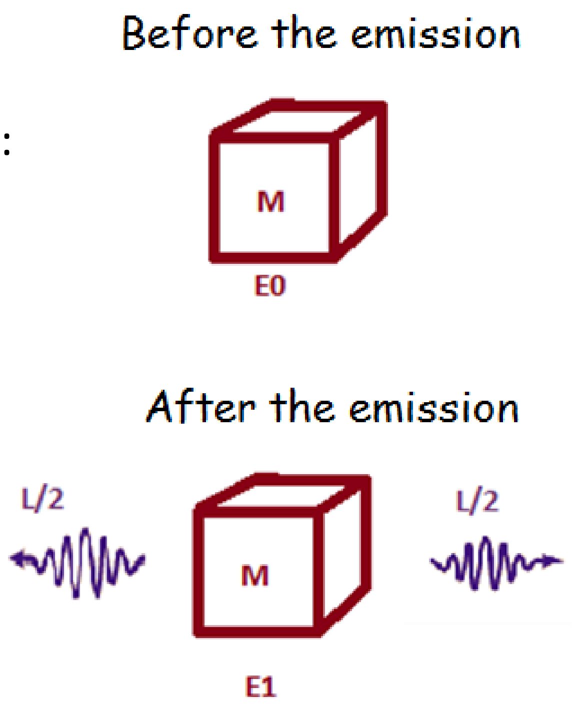
For a technical introduction to relativity check the following document:
https://web.stanford.edu/~oas/SI/SRGR/notes/srHarris.pdf

An unmoving body of mass equal to M emits two light rays with equal energies and opposite directions. An observer O is standing at rest with respect to the body. Due to conservation of momentum, the body remains at rest after the emission.

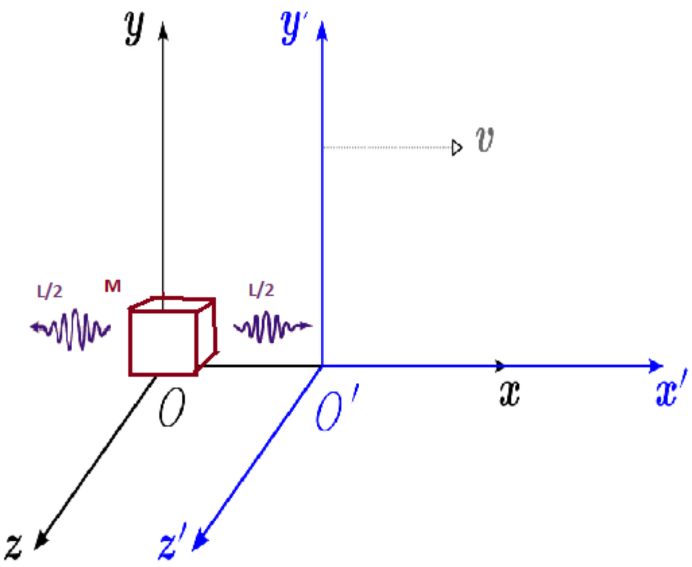
The rest energy of the body was Eo before the emission.
After the emission the new rest energy of the body becomes E1.
Each ray has energy equal to $\frac{L}{2}$.

Applying the conservation of energy theorem we have:

$$E_o = E_1 + \frac{L}{2} + \frac{L}{2} \qquad (1)$$



Another observer O’ moves with speed equal to u along the x-axis with respect to observer O as depicted on the picture below.



According to O’ the mass M will have total energy equal to Ho before the emission and energy equal to H1 after the emission of the rays.
The mass M will be moving with constant speed equal to -u before and after the emission of the light rays. According to the Lorentz transformations for energy and momentum, the energy of the rays in the O’ system will be:

$$\frac{L}{2}\frac{1-\frac{u}{c}}{\sqrt{1-(\frac{u^2}{c^2})}}$$ (ray to the right) και $$\frac{L}{2}\frac{1+\frac{u}{c}}{\sqrt{1-(\frac{u^2}{c^2})}}$$ (ray to the left)

The moving observer, O', applying the conservation of energy will find:

$$H_o = H_1 + \frac{L}{2}\frac{1-\frac{u}{c}}{\sqrt{1-(\frac{u^2}{c^2})}} + \frac{L}{2}\frac{1+\frac{u}{c}}{\sqrt{1-(\frac{u^2}{c^2})}} \rightarrow$$

$$\boxed{H_o = H_1 + \frac{L}{\sqrt{1-(\frac{u^2}{c^2})}}} \quad (2)$$

If we subtract eq. (1) from eq. (2):

$$H_o - E_o = H_1 - E_1 + \frac{L}{\sqrt{1-(\frac{u^2}{c^2})}} - L$$

Kinetic Energy of the body before the emission with respect to O'

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Since u<<c, we can expand the $\frac{1}{\sqrt{1-(\frac{u^2}{c^2})}}$ term in Taylor series. It will be equal to: $1+\frac{u^2}{c^2}$

Thus: $K_0 = K_1 + \frac{1}{2}(\frac{L}{c^2})v^2$

The body's kinetic energy with respect to O' is given by the formula: $K = \frac{1}{2}mv^2$

According to our original assumption, the speed of the body with respect to O' remains the same. Therefore the only variable that may change in order to have different kinetic energies before and after the emission is its mass. If we note mo as the mass before and m1 the mass after the emission:

$$\frac{1}{2}m_0v^2 = \frac{1}{2}m_1v^2 + \frac{1}{2}(\frac{L}{c^2})v^2$$

With some minor manipulations, the equation becomes:

$$m_0 = m_1 + \frac{L}{c^2} \rightarrow \boxed{L = (m_0 - m_1)c^2}$$

The total energy L emitted in the form of radiation leads to a reduction of the body's mass by a factor equal to

$$\boxed{\Delta m = \frac{L}{c^2}}$$

Practical implications of

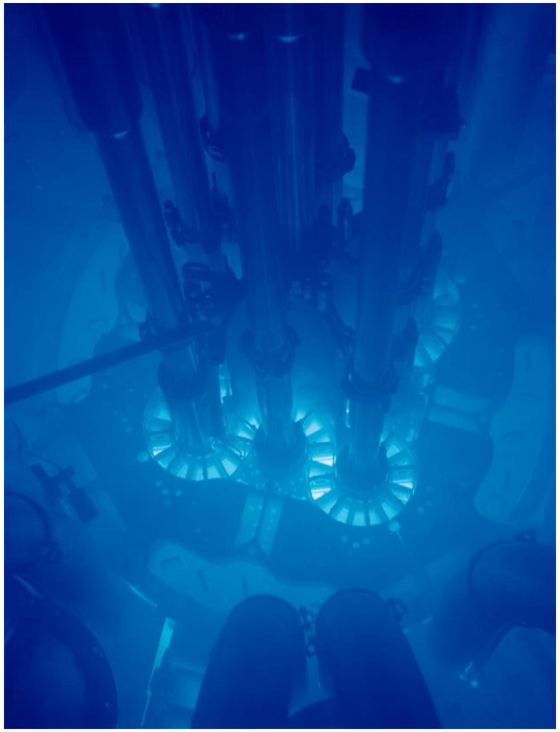
$$E = mc^2$$

Implication Nr 1:

Όduring a physical process in which energy is emitted, the mass of the system is reduced by a factor:

$$\Delta M = \frac{E}{c^2}$$

Nuclear Fusion → Nuclear Reactors:

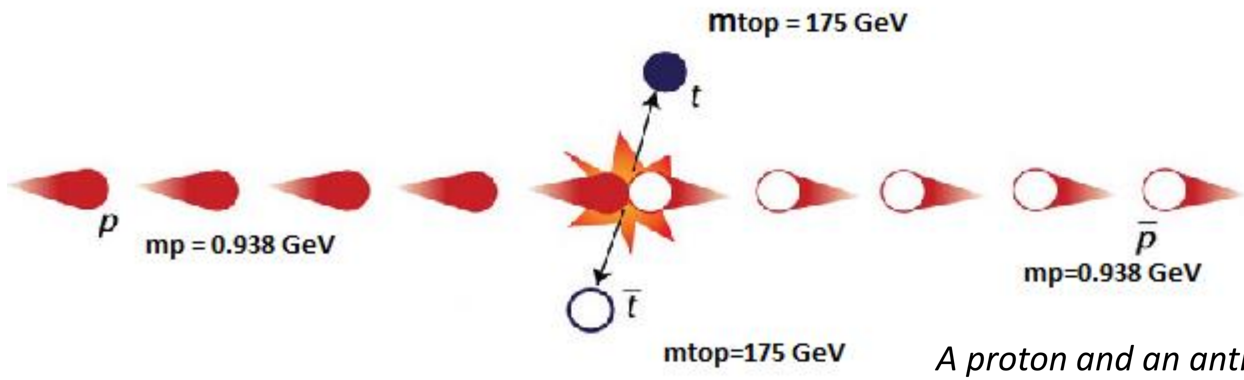


During nuclear fission, a part of the fissile nucleus's mass transforms into energy after the reaction!

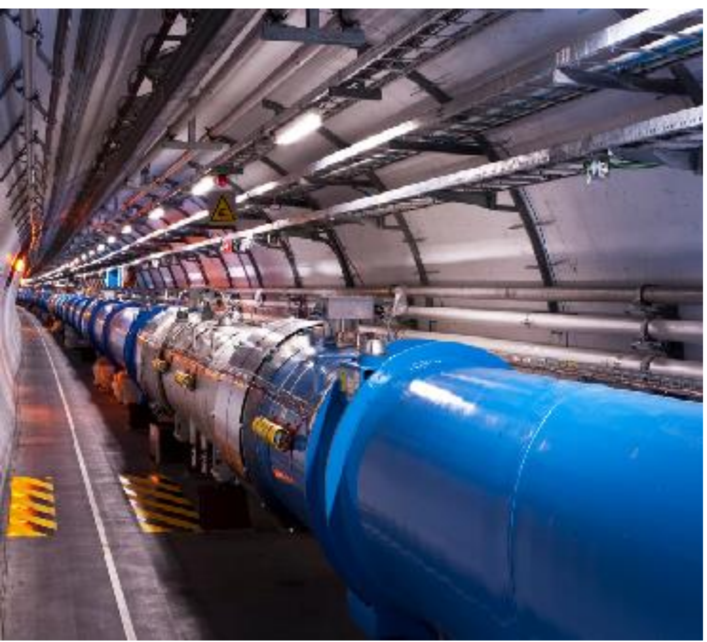
The orders of magnitude are tremendous!
Consider the following: If every month you need a barrel of gasoline for your car, using instead the energy produced by the same amount of fissile Uranium, would provide you with energy for the next 19,000 years!!!

Implication Nr 2:

If the energy produced by a suitable elementary particle interaction is equal or larger than the mass of another elementary particle, then this new elementary particle will be created even though it didn't exist before:



A proton and an antiproton collide and produce a top and an anti-top quark.



This process is utilized in particle accelerators and colliders. These machines accelerate particles such as electrons, protons, heavy ions and collide them either among themselves or with a stationary target with enough energy that will be able to produce new heavier particles.