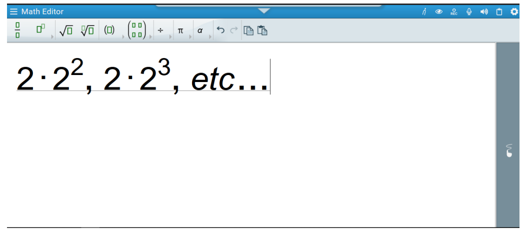
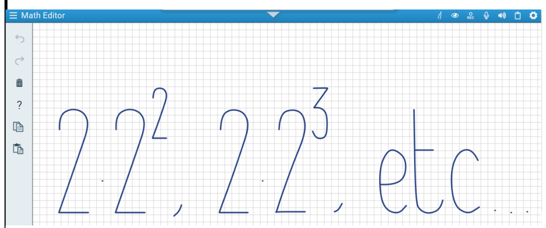
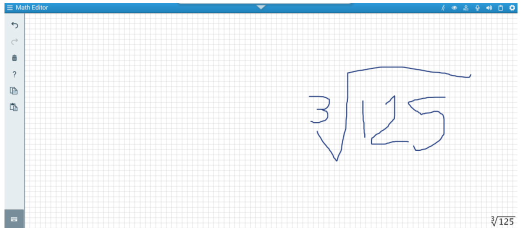
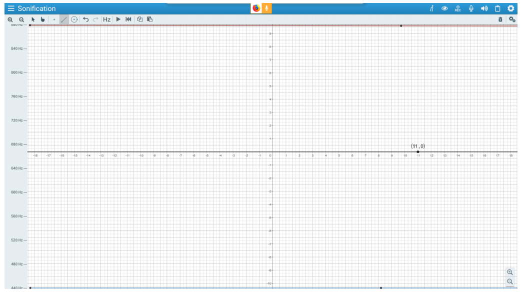
Title:	**Defining the octave**		
Keywords:	Progressions, Frequency, Tempered Scale, Diatonic Scale. Pythagoras		
Short Description:	In this scenario students observe the differences between tempered and natural diatonic scale. This is inspired by a similar exercise found in the upper-secondary book of Mathematics.		
Lesson Plans included:	-Define geometrical progressions -Graphical representation of the sequence -Geometrical comparison between progressional and proportional division of the canon	Date:	10/5/2018
Educational Objectives:	Define the ratio of geometric progressions. Compare the graph plots between two progressions. Understand the similarities between the Division of Sectio Canonis and the tempered scale	Estimated Duration:	7 hrs
Author(s):	Demetrios Papadimitriou	Age Group:	15-17 (14-16)
Contributor(s):	P. Stergiopoulos	Language:	English
Status:	Final	Difficulty Level:	Medium-Advanced
Dissemination level:	Public	Special Needs Addressed:	No

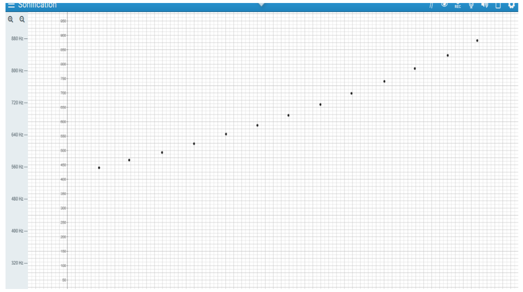
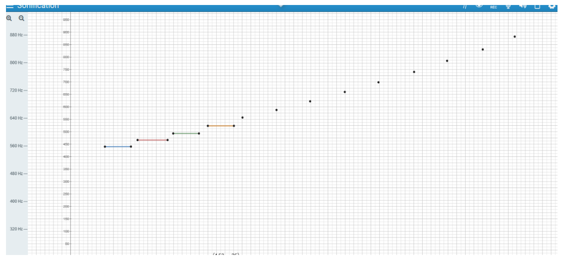
Description: This scenario is focused on presenting the differences between the tempered and the diatonic scale.
Students observe the differences between the two notes of an octave interval namely A'' on 440 Hz and A'''.
They produce a progression of the 13 consecutive terms of the geometrical progression in between the doubled frequency. They finally observe the differences between tempered and diatonic scale in ratios 1/1, 8/9, 4/5, 3/4, 2/3, 3/5, 8/15, 1/2.

E: Engineering/Technology, S: Science/Mathematics, M: Music

Phases	Field	Time	Description	Activity	Remarks
Imagine	S	1	Let's find out how a mathematical progression is related to music	Teacher introduces geometrical progressions. e.g.: 3, 3*2, 3*2*2, 3*2*2*2,... Ratio is number 2 since 3*2 = 6 3*2*2 = 12 3*2*2*2 = 24 ... etc and 12/6=2, 24/12=2 ... etc Question: If we know the value of a certain term within a progression, can we define the terms preceeding it? The answer is yes. e.g.: 1, ? , ? , 125	Teacher encourages students to start writing the progressions using math editor.  *Figure 1: Using the Math editor to introduce geometrical progressions*
				Since **first term** is 1 and r=ratio then it can be expressed as $1*r^{0}$. **Second term** of the sequence is $1*r^1$, so the 3rd term will be $1*r^2$. If **fourth term** is 125 then we have $1*r^3=125$. That means r is equal to the cube root of 125 which is 5.	Teacher encourages students to start writing the progressions using handwriting methods.  *Figure 3: Using the Math editor with hand-writing recognition*

				 *Figure 2: Using the handwrite recognition* Consequently **third term** is $1*5^2=25$ and the **second term** is $1*5^1=5$ So it is 1, 5 , 25 , 125 Question: Can this sequence be implemented in the twelve-tone music scale?	Music teacher introduces the twelve-tone scale
Engage	S, M	1	Exploring the twelve-tone scale	Teacher asks students to compose the terms of the musical scale including the octave (12+1):	Teacher helps the students express the sequence. The first term is equal to any note and the last one is equal to its octave. E.g.: We take A=440 Hz as the first term and the thirteenth term represents the octave A' = 880 Hz Music teacher plays the twelve-note scale in a tuned instrument and students use the tuner to observe the

				$1 = r^0$, $1*r^1$, $1*r^2$, $1*r^3$, $1*r^4$, $1*r^5$, $1*r^6$, $1*r^7$, $1*r^8$, $1*r^9$, $1*r^{10}$, $1*r^{11}$, $1*r^{12}$  *Figure 4: Producing 440Hz and 880Hz pitch at the Sonification environment*	frequences.
Create	S,M	1	Defining the frequencies	Students observe that the octave from the fundamental note of the scale has double frequency (440*2=880). So the first term of the sequence is equal to 440 and the last one is equal to 880, thus: • for A we have $440*r^0=440*1=440$ • for A' we have $440*r^{12}=880$ Consequently: • for A# we should have: $440*2^{1/12} \simeq$ **466**,xx Hz • for B we should have: $440*2^{2/12} \simeq$ **493**,xx Hz	Teacher encourages students to put the resulting points on the sonification tool.

					 *Figure 5: Producing the steps for each note of the octave*  *Figure 6: Listening to the notes* Students play the result by drawing lines at each point.

<table>
<tr>
<td>Analyze</td>
<td>S</td>
<td>1</td>
<td>Analyzing the results</td>
<td>They produce a table and write down their results. An example of the table is given below:

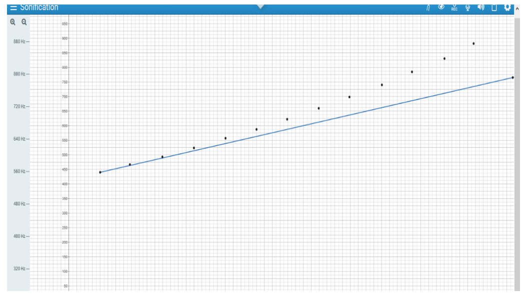
<table>
<tr><td colspan="2">Table of observations</td></tr>
<tr><td>Note</td><td></td></tr>
<tr><td>Resulting Frequency</td><td></td></tr>
</table>
</td>
<td>

Figure 7: Observing the geometrical curve

Teacher helps students identify the geometrical figure that occurs from the points by drawing a ray.</td>
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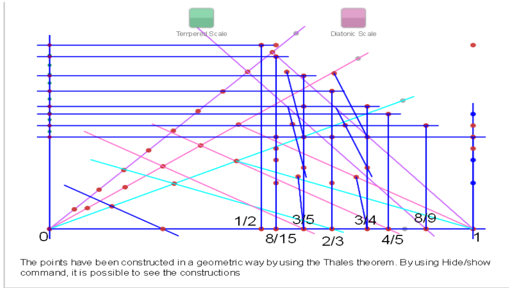
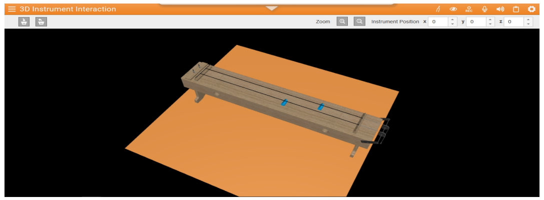
Reflect	S	1	Does your findings comply with the proportions of Thales' intercept theorem?	Students compare their results with the measurements taken by the Thales's intercept theorem. Question: Does the proportions for 1 , 8/9, 4/5, 3/4, 2/3 , 3/5, 8/15, ½ correspond to your frequency measurements? At which terms of the sequence do they do?	The results may be studied by following the steps from the “Let's hear the Thales' theorem” scenario. A summary of the results can be observed through the following Cabrilog file:  *Figure 8: Observing the differences between the two divisions*

Communicate	M	1	Produce a simple melody to be playd in the two systems of dividing the canon.	Students produce a simple melody and then they are divided in two groups that play the melody in sequence Group A plays the melody in the first way of tuning. Group B plays the melody according to the proportions implied by the above file.  *Figure 9: The 3D performance environment producing one note of the melody.*	Students with the help of their music teacher produce a sequence of melody played by four or more computers in sequence. An event in class using live performance can be arranged.