

Mathematics

Information about the Scenario

Curriculum and country:

Link of the current activity to the curriculum:

Country: Class: Grade:

Topic:

Objectives (Max 100 words):

Description of the learning objectives

Students learn that acceleration in space is a 3-dimensional vector.
Students formulate the motion equation of Newton's law with 3-dimensional vectors.
Students use their mobile phones for the measurement and display of acceleration and acceleration of gravity.

Materials (Max 100 words):

Which resources and materials (software, hardware) are needed?

Hardware: Mobile phone, classroom with beamer
Software: app "phyphox", available for iOS and Android mobiles
Remark: if students are not allowed to use their mobiles during lessons the teacher has to use his one mobile and display the screen of the mobile via remote control on a PC/beamer. Details how to use remote control at <https://phyphox.org/remote-control/>

Spatial concepts, skills and abilities:

Which spatial concepts and skills are covered by the activity?

Spatial concepts:

Primitives:	Identity/Name <input type="checkbox"/>	Location <input type="checkbox"/>	Space/Time <input checked="" type="checkbox"/>	
Simple:	Distance <input type="checkbox"/>	Direction <input checked="" type="checkbox"/>	Connectivity <input type="checkbox"/>	Movement <input checked="" type="checkbox"/>
	Boundary <input type="checkbox"/>	Shape/Area <input type="checkbox"/>	Adjacency <input type="checkbox"/>	
Difficult:	Overlay <input type="checkbox"/>	Buffer <input type="checkbox"/>	Topology <input type="checkbox"/>	Coordinate <input checked="" type="checkbox"/>
	Map <input type="checkbox"/>	Scale <input checked="" type="checkbox"/>	Shortest Path <input type="checkbox"/>	Navigation <input type="checkbox"/>
	Surface <input type="checkbox"/>	Slope/Gradient <input type="checkbox"/>	Aspect <input type="checkbox"/>	Contour <input type="checkbox"/>
Complex:	Interpolation <input type="checkbox"/>	Map Projection <input type="checkbox"/>	Spatial Dependency <input type="checkbox"/>	
Other:	<input type="text"/>			

Spatial skills:

- Map literacy
- Navigation/orientation
- Estimating distances and directions
- Recognizing and understanding patterns/Understand and identify models of spatial organization
- Select an ideal location based on the given spatial features
- Visualization
- Understand and identify spatial correlations/ dependencies
- Categorize spatial entities/ geographic features and identify hierarchies
- Compare spatial entities and draw analogies among them
- Identify/determine connections/relations
- Understanding scale in space and time
- Delineation of spatial regions/ zones based on given features/ properties

Short Description

Navigation/orientation: Finding one's way in unfamiliar environments, interpreting and giving walking and driving directions.

Estimating distances and directions: Measure paths, weighted distances, angles.

Map literacy: Using, interpreting/understanding, learning from, and communicating acquired spatial knowledge from maps, comprehension of geographic features represented as points, lines, or polygons.

Recognizing and understanding patterns/Understand and identify models of spatial organization. Delineation of spatial regions/zones based on given features/properties: Regionalization processes, pattern recognition and clustering identification in the 2d and/or the 3d world.

Select an ideal location based on the given spatial features: Single or multi-criteria siting and optimal areas identification.

Visualization: Visualizing spatial entities from written/oral verbal descriptions, from their 2d or graphical representations or through mental transformations; such as axis rotation or perspective taking.

Understand and identify spatial correlations/ dependencies: The ability to realize, identify and explain patterns, clusters and relevant spatial dependencies.

Categorize spatial entities/geographic features and identify hierarchies: Identify the hierarchical form of data and gradients between spatial entities.

Compare spatial entities and draw analogies among them: Calculate and compare different geometric objects' shapes, area and, boundaries.

Identify/determine connections/relations: The ability to identify links and common characteristics among spatial entities and between humans and spatial entities.

Understanding scale in space and time: The understanding of changes/transitions through space and time for different spatio-temporal scales.

Geospatial concepts and spatial abilities documentation (see Section 3.2):

http://www.gosteam.eu/wp-content/uploads/2021/05/GOSTEAM_IO1_A1_final.pdf

Description of the activity in detail

Classroom activities

Engage:

Using your mobile, you have learnt that your device knows its orientation in space e.g. to change from landscape format to portrait format automatically dependent from the orientation of your mobile. How the recognition of the orientation in 3-dimensional space is possible?

Explain:

Mobile devices use built in acceleration sensors, which measure the acceleration of the mobile as 3-dimensional vector $\vec{a} = (a_x, a_y, a_z)$. The measured values of this vector can be displayed by the app "phyphox". Take your mobiles, install the app "phyphox", start the app and choose in the list "Raw Sensors" the experiment "Acceleration (without g)", which gives you access to the raw data from the linear accelerometer. Start the experiment with the play button and change the orientation of your mobile in your hand. If you choose "Graph" in the menu of the experiment you see graphs of the acceleration in x, y and z direction as function of the time, if you choose "Multi" in the menu, values of the acceleration in m/s^2 are displayed. (see screenshots in figure 1)



Figure 1: Screenshots from the app "phyphox", Manfred Lohr

The interpretation of the values displayed explains positive values of the acceleration as increase of speed per time and negative values as decrease of speed per time (deceleration).

Explore:

You found out, that the result of measurements with your mobile is the acceleration vector $\vec{a} = (a_x, a_y, a_z)$. Try to find out in which directions the x-, y- and z-axis point. Put your device on the table and move it only in one direction to find out, how the coordinate system is aligned.

The correct interpretation of your experiments is that the x axis points to the right while looking at the screen in portrait (vertical) orientation, the y axis points upwards along the long side of the phone and the z axis is perpendicular to the screen, pointing out of it. (see figure 2)

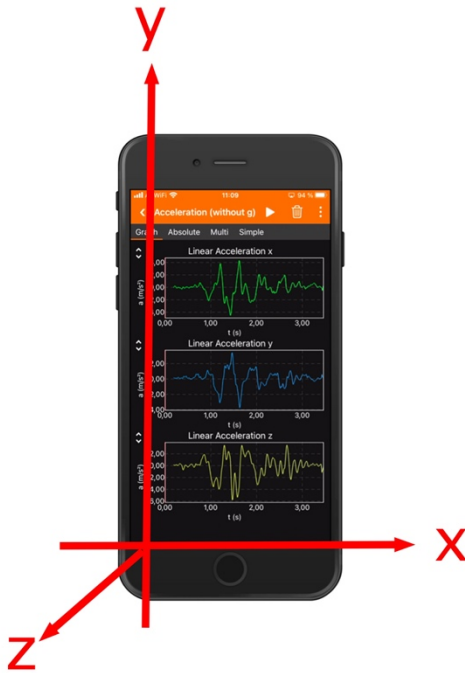


Figure 2: axes of the coordinate system, Manfred Lohr

Task 1: Use the 1st Newton's law (motion-equation) $\vec{F} = m \cdot \vec{a}$ for the calculation of the maximal force, which is acting on your mobile device, when you shake it in your hand. Determine the mass of your device and use the formula with the x, y and z components:

$$\vec{F} = (F_x, F_y, F_z) = m \cdot (a_x, a_y, a_z)$$

- (1) Mass m of the mobile = 0,2 kg, $(a_x, a_y, a_z) = (40, -5, 35)$
- (2) $\vec{F} = m \cdot (a_x, a_y, a_z) = 0,2 \cdot (40, -5, 35) = (8, -1, 7) \text{Newton}$

Task 2: Use the "phyphox" experiment "Acceleration with g", start the measurement and place the mobile on a horizontal table. Consider, why the accelerometer displays 0 for the x- and y-axis, but for the z-axis a value near 10 m/s^2 although the mobile device at rest. (see screenshot in figure 3)

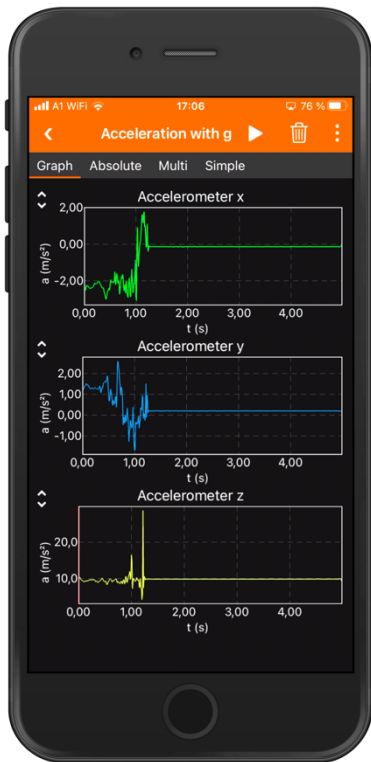


Figure 3: acceleration with g, mobile at rest, Manfred Lohr

If you choose “Multi” in the menu of the app, you realise that the value of the acceleration of the z-axis is $a_z = 9,8 \text{ m/s}^2$.

This acceleration is caused by the force of gravity of the earth, which was explored by Isaac Newton and explained in his law of gravity:

$$F = G \frac{m_1 \cdot m_2}{r^2}$$

In this formula G is the constant of gravitation ($G = 6,67 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$), m_1 and m_2 are two masses and r is the distance between the center of these masses.

Use your mobile, start the animation “Gravity Force Lab” (Link:

https://phet.colorado.edu/sims/html/gravity-force-lab/latest/gravity-force-lab_en.html) and

explore the law of gravity. (see screenshot in figure 4)

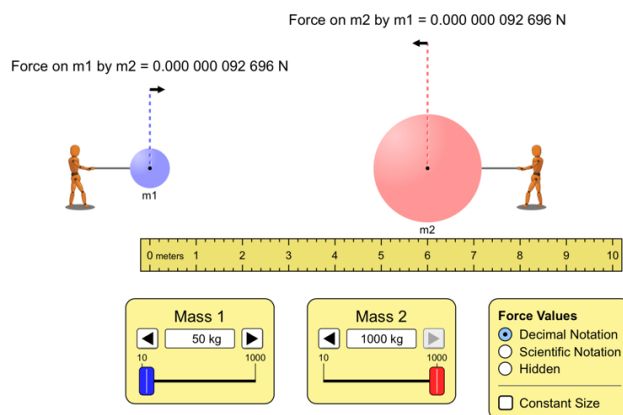


Figure 4: Animation "Gravity Force Lab", source <https://phet.colorado.edu>

Now you know that the value of a_z is caused by the gravity of the earth. The following steps explain, why the value of a_z is $9,8 \text{ m/s}^2$:

- (1) $F = m_1 \cdot a$... gravity force acting on the mobile phone with the mass m_1
- (2) $F = G \frac{m_1 \cdot m_2}{r^2}$... gravity force explained by Newton's law of gravity
- (3) $m_1 \cdot a = G \frac{m_1 \cdot m_2}{r^2}$... m_2 is the mass of the earth, r is distance between m_1 and m_2 , which is the radius of the earth if we assume that the earth is a perfect sphere
- (4) $a = G \frac{m_2}{r^2}$... $G = 6,67 \cdot 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}$, $m_2 = 5,98 \cdot 10^{24} \text{ kg}$, $r = 6371 \cdot 10^3 \text{ m}$
- (5) $a = 9,83 \text{ m/s}^2$

The acceleration caused by the gravity of the earth is called "acceleration of gravity g ". If you consider equation (4) you recognize that the acceleration of gravity depends on the distance between m_1 and m_2 . Start the following animation "Gravitational field" on you mobile and explore the direction of g and search how g depends on the height h of the mass m_1 above the surface of the earth:

https://www.vascak.cz/data/android/physicsatschool/template.php?s=gp_centralni_pole&l=en

(in the animation g is named K_h , the constant of gravitation is named κ and the mass and radius of the earth are M and R , see screenshot in figure 5)

$$M = 5,98 \cdot 10^{24} \text{ kg}$$

$$R = 6371 \cdot 10^3 \text{ m}$$

$$\kappa = 6,67 \cdot 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}$$

$$h = 2476 \text{ km}$$

$$K_h = \frac{\kappa M}{(R + h)^2}$$

$$K_h \doteq 5,10 \text{ N} \cdot \text{kg}^{-1}$$

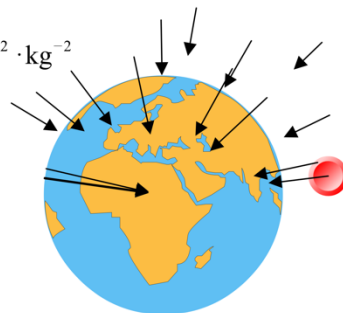


Figure 5: Gravitational Field, source <https://www.vascak.cz>

In the considerations you did before it is assumed that the earth is a perfect sphere. But in reality the earth shows an ellipticity because of the rotation of the earth. Therefore the radius of the earth is shortened on the poles and lengthened on the equator. Considering formula (4) you realise that this fact causes a bigger value of g on the poles and a smaller value on the equator.

But the flattening of the earth is not perfect either. There are further deviations because the distribution of the earth's masses is spatially non-uniform. The representation of the gravitational deviations of the earth's shape compared to the regular ellipsoidal surface has become known as the "Potsdam potato". This figure is based on a gravity model calculated at the German Research Centre for Geosciences in Potsdam, Germany. The following animation shows this "Potsdam Gravity Potato": move the mouse pointer to one of the speeds (Bilder/s)

on the right side, as long as the mouse pointer remains on this field, the earth rotates continuously at the selected speed.

<http://www-app2.gfz-potsdam.de/sec13/animated-potato-d-cms.html>

Try to find reasons for the importance of the knowledge of the exact value of gravitational acceleration.

Exact data of the gravity are crucial for ocean circulation and sea-level change, both of which are affected by climate change. The data help to better understand processes occurring inside the Earth which are linked to volcanoes and earthquakes.

Online activities

Prepare and carry out two experiments with your mobile phone and the app phyphox. Start the experiment “acceleration with g” in the app “phyphox”. Stand in front of your bed and hold your mobile phone as high as possible above the mattress. Hold the mobile in a horizontal position for a few seconds and then let it fall. Take your mobile, stop the measurement and describe the graphs of the z-axis displayed in “Acceleration z”. If you tap on the left beside the accelerometer graph z you view the results only from the z-axis. Make screenshots of your measurement and share it in a videoconference software to your classmate and teacher. Describe the result shown in the graph. The result should be similar as shown in figure 6.



Figure 6: acceleration during free fall, Manfred Lohr

You recognize that during you hold you mobile stable in a horizontal position the acceleration is about zero on the x- and y-axis and about 10 m/s^2 on the z-axis caused by the gravity of the earth. But during the free fall of your mobile the acceleration on the z-axis decreases to zero! This phenomenon is called “zero gravity” or “micro gravity”. Zero gravity occurs because during free fall there is no other directly acting counter force affecting the falling mobile.

Zero gravity can be experienced during so called parabolic flights – use the website <https://www.gozerog.com/home/> and report about your findings during the video conference.

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